JOURNAL OF AIRCRAFT
Vol. 40, No. 2, March–April 2003

From Fluttering Wings to Flapping Flight: The Energy Connection

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A new paradigm is presented for understanding flutter and flapping flight. The framework for the investigation is based on conservation of energy. Energy produced, energy lost and/or work done due to structural vibration, aerodynamic wake, and propulsion are taken into account. It is shown that there exist three types of modes in an aeroelastic system, namely 1) unstable mode producing drag (flutter mode), 2) stable mode producing drag, and 3) stable mode producing thrust (flapping flight mode). The type of mode can be determined from the mode shape for a given reduced frequency. The regions (in the modal vector space) corresponding to the three types of modes are presented for a two-dimensional airfoil. It is shown that the flutter region is separate from the thrust-producing region and the boundaries of these regions are tangential to one another at a common neutral point. The neutral point corresponds to no energy transfer between the energy sources. The efficiency of flapping flight and occurrence of flutter is further investigated. Finally, the possibility of a limit-cycle oscillations due to constant thrust flight is presented.

Introduction

THE objective of the present work is to investigate the energy transfer pathways that form the basis for aeroelastic flutter and flapping flight. The focus is to study both phenomena in a common framework, compare them for similarities, and learn from overall results.

Most of the research in the available literature has focused on either wing flutter or flapping flight. There has not been much effort to integrate these two apparently different but actually quite similar phenomena. Most of the aeroelastic flutter research has been conducted using various aerodynamic and structural models without much regard to the energy aspects of the problems (some exceptions include work by Nissim,[1] Horikawa and Dowell,[2] and Bendiksen[3]). On the other hand, the research on flapping flight has almost always used energy concepts along with various aerodynamic modeling techniques (for example, Garrick,[4] DeLaurier,[5] and Pendaries[6]). Most of the flapping flight research though has been conducted without considering the structural part in detail, in part because much of the research was conducted on rigid bodies in which motion could be easily prescribed.

A complete view of the energy transfer mechanisms responsible for various aeroelastic phenomena is required. It should be able to explain wing flutter as well as flapping flight and is expected to be helpful in further understanding both of those. In addition, it is expected that such an energy-based perspective would provide new tools for design and analysis of flexible flapping wings, energy-based aeroelastic control synthesis, and integrated aircraft aeroelastic analysis. This forms the motivation of the present research.

Energy Considerations

The prevalent perspective (present in most textbooks and research papers explaining flutter) on energy flow during aeroelastic instability can be summarized as shown in Fig. 1. From this view, it is easy to see that for a fluttering mode the energy is transferred from the flow to the structure, leading to increase in oscillation amplitude. For a stable mode, the energy is transferred from the structure to the flow, and thus, the energy of the structure decreases leading to decrease in oscillation amplitude. The amount and direction of this energy flow is dependent on the phase difference between the generalized force and corresponding generalized speed for the particular mode.[7] Though accurate, this view of energy flow is incomplete because all of the energy sources/sinks are not included. Such an energy transfer mechanism seems to indicate incorrectly that at flutter, because the energy is transferred from the flow to the structure, the energy of the flow is decreasing. In fact, it can be easily shown by calculating the energy of the wake that, even at flutter, energy is released into the flow and the fluid energy is increasing. (Exception should be made here for flutter in a wind tunnel, where the flow energy does decrease.) Thus, at flutter the energy of the structure as well as the flow is increasing, which cannot be explained by Fig. 1.

The present approach is shown in Fig. 1. It provides a complete picture of the energy flow mechanisms in flutter by including all of the energy sources/ sinks of the complete aircraft. This approach is regularly used in flapping flight analysis (see the classic work by Garrick[4]). It includes three energy sources/sinks: 1) structure, which denotes the energy required to maintain a constant level of oscillation in the given mode, 2) wake, which is the energy lost in the fluid as shed vortices, and 3) propulsion, which is the energy required by the engine to maintain a given speed while the structure is oscillating.

All of the energies mentioned can be calculated for a particular mode shape and reduced frequency. Depending on the direction of energy flow, from or into, particular energy source/sink, one can determine the characteristics of the mode. For the energy source/sink denoted by structure, the direction of energy flow indicates the work done by or on the structure. If the structural energy is increasing, it indicates an unstable (flutter) mode, whereas decreasing structural energy (energy flowing from the structure into the other energy sources/sinks) indicates an aerodynamically damped mode. For the energy source/sink denoted by propulsion, increase in propulsive energy (flow of energy into propulsion) indicates that the mode is producing thrust, whereas decreasing propulsive energy indicates that the oscillations in that mode lead to drag. Finally, wake energy is the kinetic energy of the fluid due to the vorticity shed during the structural oscillations in an otherwise stationary flow. Thus, this energy is constantly increasing.

The interactions between the three energy sources/sinks lead to various kinds of aeroelastic modes, as shown in Fig. 2. Case 1 (Fig. 2a) denotes a fluttering mode. Here we see that for flutter

Presented as Paper 2001-1460 at the 42nd Structures, Structural Dynamics and Materials Conference, Seattle, WA, 16–19 April 2001; received 15 July 2001; revision received 18 June 2002; accepted for publication 20 June 2002.

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to occur the energy of the structure should increase. The energy of the wake is also increasing during flutter due to shed vortices. Thus, the energy required to support the structural instability has to come from the propulsion. This additional propulsive energy is introduced into the system via the increase in the drag associated with the flutter mode. Because of the increase in drag, more thrust is required to maintain the flight speed, and thus, additional propulsive energy is introduced into the system. This propulsive energy introduced to maintain the airspeed is transferred to the structure causing flutter. This conclusion has not been reported in the literature and provides the energy basis for flutter.

If the mode is stable (aerodynamically damped), the energy is transferred from the structure to the other energy sources/sinks. Because the energy is always released in the wake, one can now have two possibilities: case 2 (Fig. 2b), where propulsive energy is also expended, that is, there is an increase in drag, or case 3 (Fig. 2c), where some of the energy put into the system via the structure is converted to propulsive energy, that is, the structural vibrations leads to thrust. Thus, case 2 denotes a damped, drag-producing mode, and case 3 denotes a damped, thrust-producing mode, the ones that are studied in flapping flight research.

Thus, the present view of the energy flow mechanisms leads to three kinds of modes, namely, 1) unstable mode with increase in drag (flutter mode), 2) stable mode with increase in drag, and 3) stable mode generating thrust (flapping flight mode). It can now be seen that there cannot be an unstable mode that generates thrust because such a mode would lead to increase in all of the energies, implying production of energy out of nothing.

It is clear from the energy perspective that the flutter mode and the flapping flight mode are just two types of the same problem. These modes, however, have a completely different energy transfer mechanism and in a sense are quite opposite to each other. In flutter, propulsive energy is transferred to the structure, whereas in thrust mode, energy introduced through the structure is converted to propulsive forces. In both cases energy is wasted through the shed wake in the fluid. However, the fluid is the medium through which this transfer of energy takes place, and the dissipation of energy into the flow is an expected loss in the energy conversion process. A simple example is now presented to highlight the energy transfer mechanisms.

Example

Let us consider as an example a two-dimensional airfoil undergoing sinusoidal oscillations in pitch and plunge. Using the work of von Kármán and Burgers,[8] Theodorsen,[9] and Garrick,[4] one can write

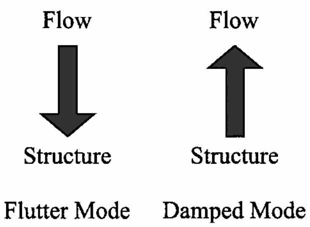

Fig. 1 Prevalent view of energy pathways in aeroelasticity.

the expression for the lift L, moment M, drag D, and circulation Γ as

$$L = 2\pi\rho b\left[(b/2)(U\dot{\alpha} + \ddot{h}) + C(k)U\left(U\alpha + \dot{h} + \tfrac{1}{2}b\dot{\alpha}\right)\right]$$

$$M = 2\pi\rho b^2\left[-(b^2/16)\ddot{\alpha} - (bU/4)\dot{\alpha} + (U/2)C(k)\left(U\alpha + \dot{h} + \tfrac{1}{2}b\dot{\alpha}\right)\right]$$

$$D = L\alpha - 2\pi\rho b\left[C(k)\left(U\alpha + \dot{h} + \tfrac{1}{2}b\dot{\alpha}\right) - \tfrac{1}{2}b\dot{\alpha}\right]^2$$

$$\Gamma = 2\pi b\left[H(k)\left(U\alpha + \dot{h} + \tfrac{1}{2}b\dot{\alpha}\right)\right] \tag{1}$$

where h and α are the midchord plunge and pitch angle about the midchord, ρ, b, and U are the air density, semichord, and airspeed, respectively, and $C(k)$ and $H(k)$ are functions of the reduced frequency $k = \omega b/U$ and can be expressed in terms of Bessel functions $[H_1^{(2)}(k)$ and $H_0^{(2)}(k)]$ as

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + iH_0^{(2)}(k)}, \qquad H(k) = \frac{1}{H_1^{(2)}(k) + iH_0^{(2)}(k)} \tag{2}$$

The expression for the change in the various energies over one oscillation can be written as

$$W_s = \int_0^T (-L\dot{h} + M\dot{\alpha})\,\mathrm{d}t, \qquad W_p = \int_0^T (-DU)\,\mathrm{d}t$$

$$W_a = \frac{1}{2}\int_0^{UT} (w_z \Delta\phi)\,\mathrm{d}x \tag{3}$$

where T is the period of oscillation, w_z is the induced velocity due to shed vorticity and $\Delta\phi$ is the potential difference between the two sides of the shed vortex sheet. W_s denotes the energy extracted from the structure (positive) to avoid increase in oscillations or the necessary external work done on the structure (negative) to maintain the oscillations, W_p denotes the propulsive energy produced due to thrust (positive) or the propulsive work done to cancel the drag (negative), and W_a denotes the kinetic energy released in the flow (positive). The three energies as defined here can be related to the energies used by Garrick in his paper on propulsion of flapping and oscillating airfoils.[4] W_s, W_p, and W_a are the same as the terms denoted by $-W$, Pv, and E in Ref. 4. Using conservation of energy, we have

$$W_s + W_p + W_a = 0 \tag{4}$$

When sinusoidal variation in pitch and plunge are assumed as

$$h = \bar{h}_0 b e^{iwt}, \qquad \alpha = \alpha_0 e^{iwt} \tag{5}$$

the rate of change of the various energies averaged over one oscillation can be written in the nondimensional form as

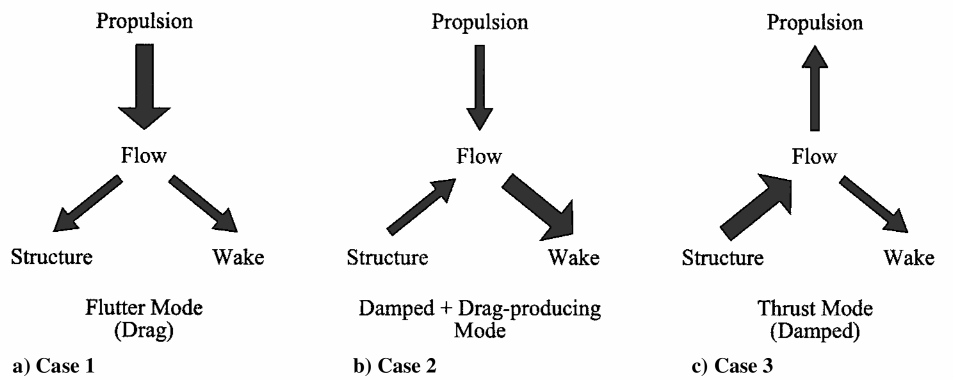

Fig. 2 Proposed perspective of energy transfer mechanisms in aeroelasticity and flapping flight.

$$\bar{W}_s = \Re\Big\{\Big[\tfrac{1}{2}\big(ik\alpha_0 - k^2\bar{h}_0\big) + C(k)\big(\alpha_0 + ik\bar{h}_0 + \tfrac{1}{2}ik\alpha_0\big)\Big](-ik\bar{h}_0)^T + \Big[\tfrac{1}{16}k^2\alpha_0 - \tfrac{1}{4}ik\alpha_0 + \tfrac{1}{2}C(k)\big(\alpha_0 + ik\bar{h}_0 + \tfrac{1}{2}ik\alpha_0\big)\Big](ik\alpha_0)^T\Big\} \tag{6}$$

$$\bar{W}_p = \Re\Big\{\Big[C(k)\big(\alpha_0 + ik\bar{h}_0 + \tfrac{1}{2}ik\alpha_0\big) - \tfrac{1}{2}ik\alpha_0\Big] \times \Big[C(k)\big(\alpha_0 + ik\bar{h}_0 + \tfrac{1}{2}ik\alpha_0\big) - \tfrac{1}{2}ik\alpha_0\Big]^T - \Big[\tfrac{1}{2}\big(ik\alpha_0 - k^2\bar{h}_0\big) + C(k)\big(\alpha_0 + ik\bar{h}_0 + \tfrac{1}{2}ik\alpha_0\big)\Big](\alpha_0)^T\Big\} \tag{7}$$

$$\bar{W}_a = [2/(\pi k)]\Re\Big\{\Big[H(k)\big(\alpha_0 + ik\bar{h}_0 + \tfrac{1}{2}ik\alpha_0\big)\Big] \times \Big[H(k)\big(\alpha_0 + ik\bar{h}_0 + \tfrac{1}{2}ik\alpha_0\big)\Big]^T\Big\} \tag{8}$$

All of the energies are averaged over one oscillation and nondimensionalized with respect to $\pi\rho b U^3$. It can be shown that the total of the three energies is always zero, that is, the energy released by one is absorbed by another. The complete fluid–structure propulsion system is thus conservative as expected.

Using the preceding expressions, one can now plot the various energies as a function of the mode shape and the reduced frequency. Figures 3, 4, and 5 show the contour plot for the structural, propulsive, and wake energies/work done respectively calculated using Eqs. (6–8). Figures 3–5 show the variation in the energies with mode shape, that is, α_0 (phase and magnitude, represented as the real and imaginary part) relative to $\bar{h}_0$ (fixed at 1). Reduced frequency of 0.25 is used. Figure 3 shows the structural energy contour plot. The zero energy contour divides the α_0 space into a stable zone and a flutter region. Thus, the system can go unstable only if one of the modes has a mode shape (phasing) that falls in the flutter region. Figure 3 is similar to that presented by Greidanus[7] and can be related to the aerodynamic energy concept of Nissim.[1] Figure 4 shows the the variation of propulsive energy (thrust/drag) with mode shape. It is seen that, for $k = 0.25$, positive propulsive energy (thrust) is possible only for a small range of α_0 relative to $\bar{h}_0$. Finally, Fig. 5 shows the aerodynamic wake energy contours. It is the energy lost in the unsteady wake. The energy dissipation is different for different mode shapes due to the change in the level of the unsteady circulation. From the expression for the circulation, one can see that for $\alpha_0 = [-ik/(1 + 0.5ik)]\bar{h}_0$ the unsteady circulation is zero and there is no shed wake. Thus, there is no dissipation of energy into the wake, and we have the wake energy going to zero for this value. This point is marked by the asterisk. Away from this point, the unsteady circulation and the wake energy increases.

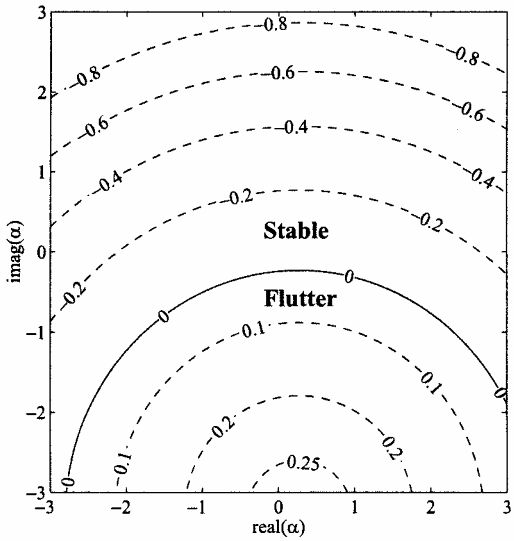

Fig. 3 Contours of structural energy/work done for k = 0.25.

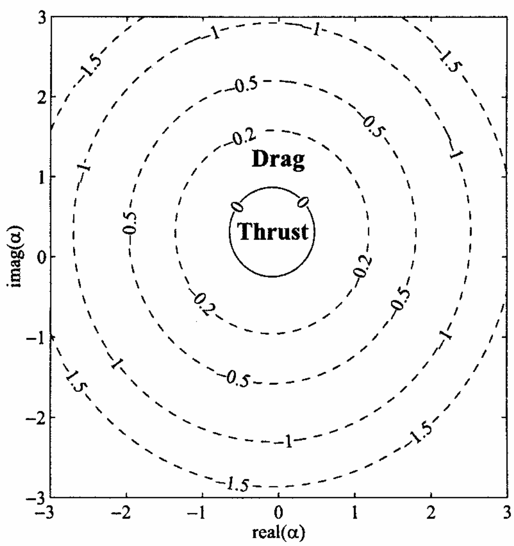

Fig. 4 Contours of propulsive work done for k = 0.25.

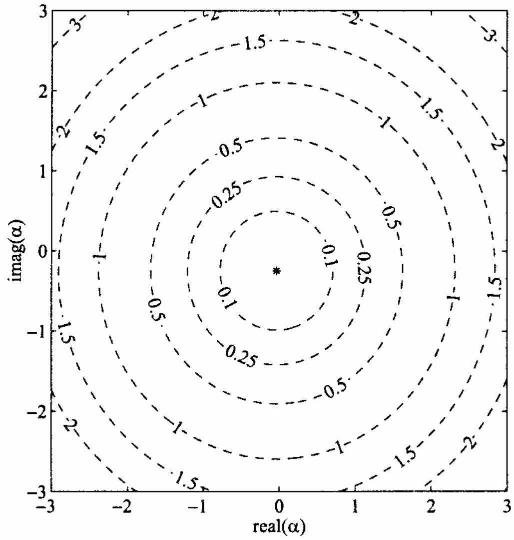

Fig. 5 Contours of kinetic energy of the wake for k = 0.25.

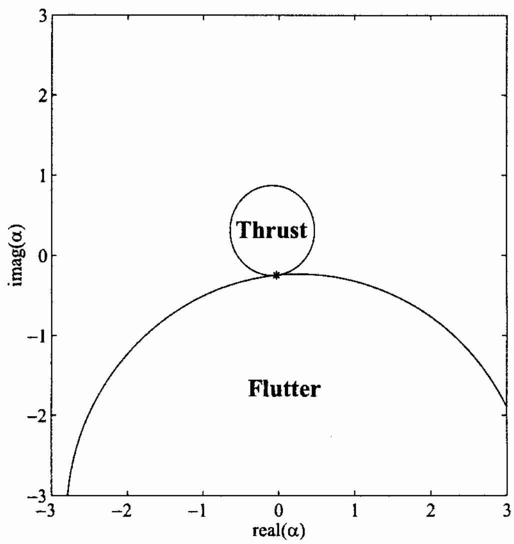

Fig. 6 Contours defining the boundary of flutter and thrust production, k = 0.25.

Now we plot the neutral (zero energy/work done) contours for the various energies on the same plot. Figure 6 shows the flutter boundary, the thrust boundary, and the zero aerodynamic wake energy point for $k = 0.25$. From Fig. 6, one can see that 1) the flutter and thrust-producing regions are separate, 2) for the present

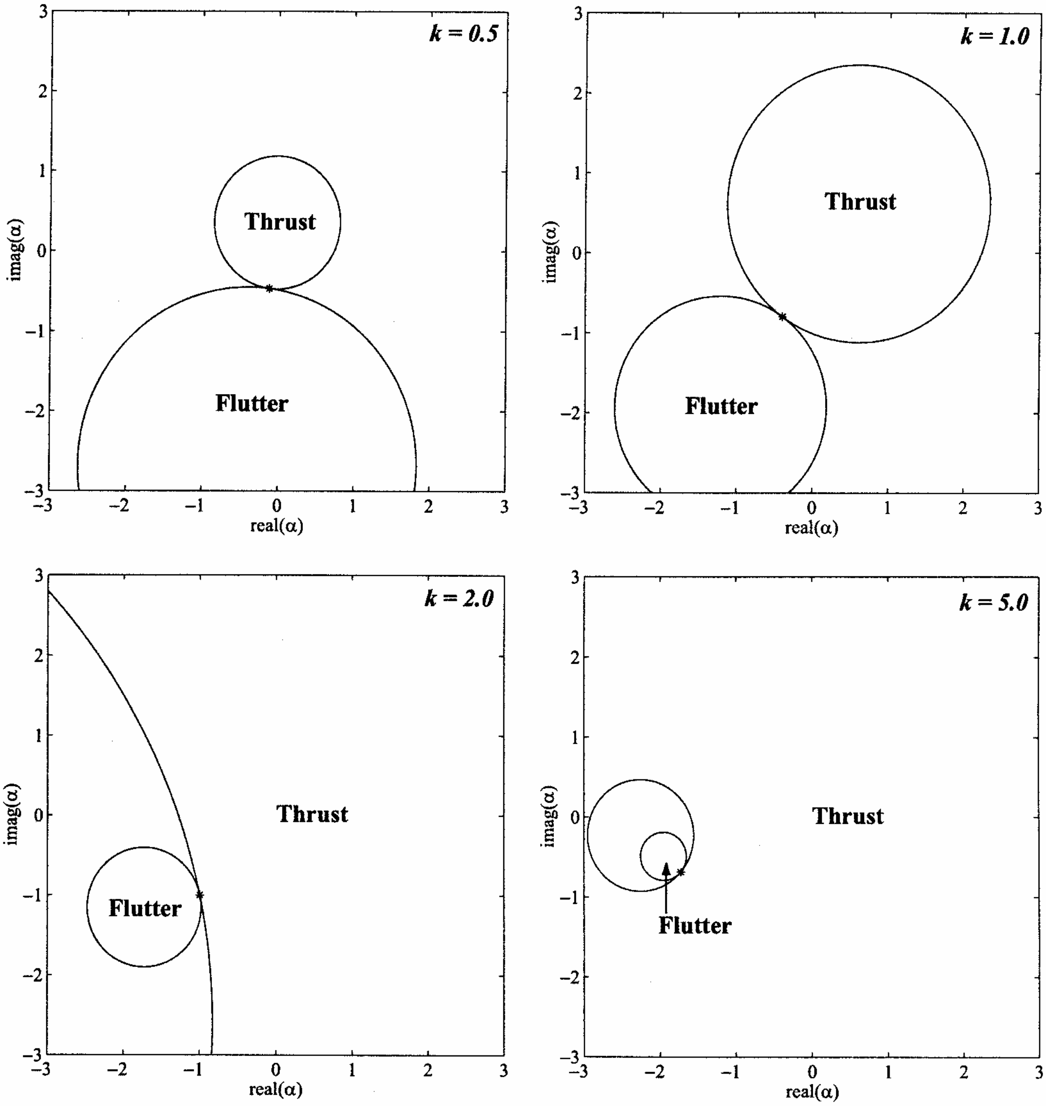

Fig. 7 Change in flutter and thrust regions with reduced frequency.

(two-dimensional airfoil) case the flutter boundary and thrust boundary are tangential to one another and touch at a common point, and 3) the point where the two curves touch also corresponds to the zero energy release in the wake. Thus, there exists a neutral point, where the rate of change of all energies is zero. Also, although flutter mode and thrust mode can never be the same, the mode shapes for the two can be arbitrarily close to one another.

The thrust boundary and flutter boundary are now plotted for various other reduced frequencies. Figure 7 shows the neutral energy contours for $k = 0.5$, 1.0, 2.0, and 5.0. It is seen that, as the reduced frequency increases, the range of mode shapes for flutter decreases, whereas the range of mode shapes for thrust increases. The decrease in flutter range is expected because an increase in k corresponds to a decrease in U, the aircraft velocity. The increase in the thrust range is also expected as shown in Ref. 4.

Thus, it is seen that a simple airfoil oscillating in incompressible flow shows the various energy transfer mechanisms as proposed in the present paper.

Energy Transfer Efficiency

As shown in the earlier section, there are regions in the modal space that lead to induced thrust. The success of flapping flight though depends on the efficiency of energy conversion. Figure 8 shows the flapping (thrust-production) efficiency defined as the ratio of propulsive energy output to the structural work done. The contours for $\bar{W}_p$ are also shown, as dotted lines. It is clear that the efficiency of thrust generation tends to 1.0 as the mode nears the neutral point. However, the magnitude of thrust generated (per unit deflection) is small near the neutral point. Thus, it is a tradeoff between the amount of thrust produced and the effciency of thrust production. From Fig. 8 it is clear that, for the present example, if one operates at the maximum $\bar{W}_p$ output level, the efficiency of energy conversion is only around 0.4–0.5. Depending on the amount of thrust required and the range of motion allowed, one can then try to find the mode shape for the best efficiency at the required thrust.

Figure 9 shows the energy transfer efficiency for the flutter mode. It is the ratio of energy transferred into the structure to the total energy generated by the propulsive power plant. Again, one gets high efficiency near the neutral point, but the strength of the instability, $\bar{W}_s$, is lower.

Actual Aeroelastic Mode

The results presented were all generated by considering just the aerodynamic characteristics of the model. It would be interesting to see how the actual modes of the system fit into this scheme.

Let us consider that the two-dimensional airfoil is connected to a base via plunge and pitch springs. The equations of motion of the aeroelastic system can be written as

$$\begin{bmatrix} m_h & s_{h\alpha} \\ s_{h\alpha} & I_\alpha \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} k_h & r_{h\alpha} \\ r_{h\alpha} & k_\alpha \end{bmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} -L \\ M \end{Bmatrix} \tag{9}$$

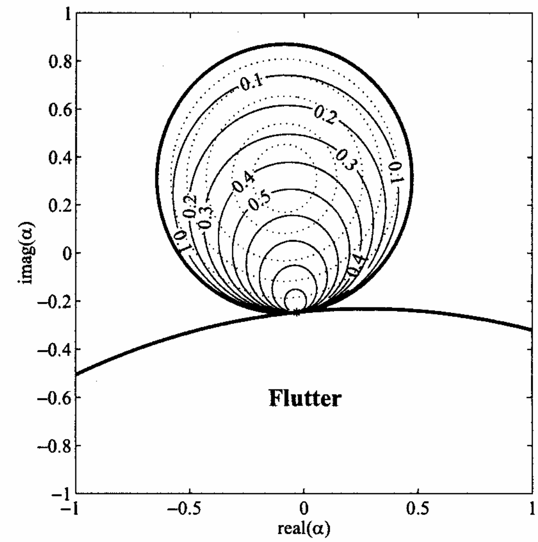

Fig. 8 Contours defining the propulsive efficiency, k = 0.25.

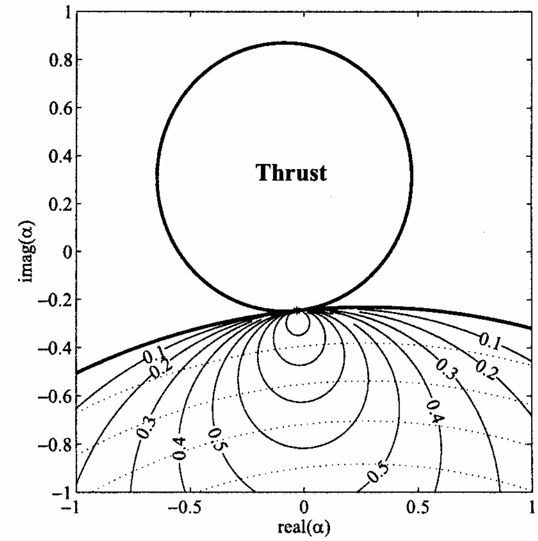

Fig. 9 Contours defining the flutter efficiency, k = 0.25.

where m_h and I_α are the airfoil mass and moment of inertia, k_h and k_α are the spring stiffinesses for plunge and pitch, $s_{h\alpha}$ is the mass offset term, and, $r_{h\alpha}$ is the coupling stiffness.

Again, assuming sinusoidal variation in the pitch and plunge as

$$h = \bar{h}_0 b e^{iwt}, \qquad \alpha = \alpha_0 e^{iwt} \tag{10}$$

one could write the equations of motion for aeroelastic analysis using the k method as

$$\frac{1+ig}{\omega^2}\begin{bmatrix}\omega_h & \omega_h x_k\\ \omega_h x_k & \omega_\alpha\sigma^2\end{bmatrix}\begin{bmatrix}\bar{h}_0\\ \alpha_0\end{bmatrix} = \left(\begin{bmatrix}1 & x_m\\ x_m & \sigma^2\end{bmatrix} + \frac{2}{\mu k^2}\right.$$
$$\left.\times\begin{bmatrix}-\left[-\dfrac{k^2}{2}+ikC(k)\right] & -\left[i\dfrac{k}{2}+C(k)\left(1+i\dfrac{k}{2}\right)\right]\\ i\dfrac{k}{2}C(k) & \dfrac{k^2}{16}-i\dfrac{k}{4}+\dfrac{C(k)}{2}\left(1+i\dfrac{k}{2}\right)\end{bmatrix}\right)$$
$$\times\begin{Bmatrix}\bar{h}_0\\ \alpha_0\end{Bmatrix} \tag{11}$$

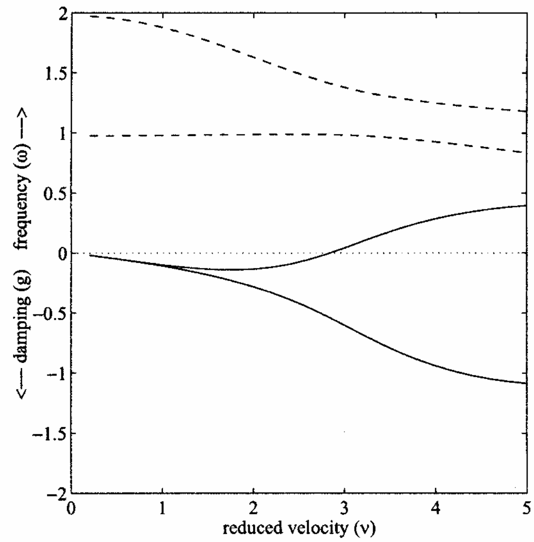

Fig. 10 V–g plot from flutter analysis.

where ω is the frequency of the mode, g is the artificial damping introduced to maintain harmonic oscillations, and the other parameters are defined as

$$\omega_h = \sqrt{k_h/m_h}, \qquad x_m = s_{h\alpha}/(m_h b), \qquad \omega_\alpha = \sqrt{k_\alpha/I_\alpha}$$
$$x_k = r_{h\alpha}/(k_h b), \qquad \sigma = \sqrt{I_\alpha\Big/\left(m_h b^2\right)}, \qquad \mu = m_h\Big/(\pi\rho b^2) \tag{12}$$

Using the preceding equations, one can find the aeroelastic modes of the system at various reduced frequencies. The k method gives the modal frequency ω, the mode shape $\{\bar{h}_0, \alpha_0\}^T$, and the artificial damping g required to maintain harmonic motion for the aeroelastic modes. The modal frequency and artificial damping are calculated for a range of reduced velocity $\nu = 1/k = U/\omega b$. The reduced velocity at which the artificial damping becomes positive gives the reduced velocity corresponding to flutter.

Figure 10 shows the change in aeroelastic modal frequency and artificial damping with increase in reduced velocity. The example uses $\omega_h = 1.0$, $\omega_\alpha = 2.0$, $x_m = 0$, $x_k = 0$, $\mu = 20$, and $\sigma = 0.5$. It is seen that one of the modes goes unstable at $\nu_F = 2.844$.

Now let us look at the same result using the new perspective. We need to compare the mode shape of the actual aeroelastic mode with that required for flutter or thrust mode. Figure 11 shows mode shapes of the actual aeroelastic modes at various reduced velocities. (Dark circle symbols denote the relative magnitude and phase of pitch with respect to plunge.) Also, plotted are the boundaries of the three possible types of mode. At low ν, only one mode is seen because the other mode (pitch-dominant) is out of the plot scale. The plunge-dominant mode is seen to be a thrust producing mode. In fact, this mode stays a thrust producing mode for all of the reduced frequencies shown. The pitch-dominant mode comes within the bounds of the plots for $\nu \geq 2.0$. It is clearly a drag-producing stable mode for low ν. As the ν increases, the mode comes closer to the flutter boundary, and as expected (from earlier flutter calculation of $\nu_F = 2.844$), it crosses into the flutter region for $2.5 < \nu < 3.0$. It moves within the flutter boundary for higher ν and seems to be approaching the neutral point.

Thus, using the energy approach, one gets more insight into the flutter problem and can predict the type of the mode from the mode shape. How a designer uses this added insight is open for discussion.

Flutter and Propulsive Energy

It is clear that when the wing flutters, the energy comes from the propulsion, that is, when the wing starts to oscillate in the flutter mode, it leads to an increase in the drag. Thus, to maintain the aircraft velocity, the thrust produced by the engine must be increased. This additional energy expenditure results in the increase in the structural

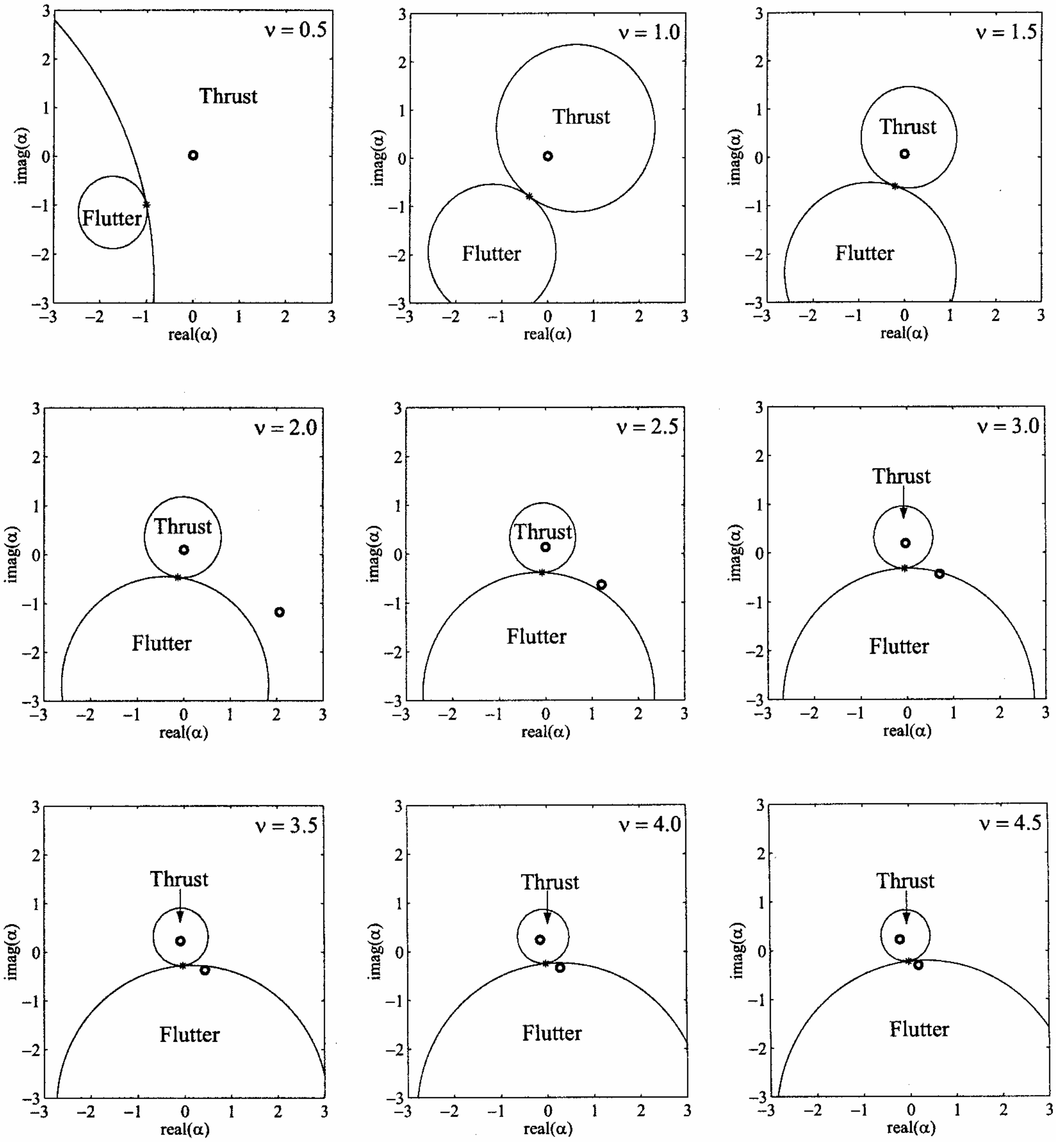

Fig. 11 Change in mode with reduced velocity ν; mode shape (relative magnitude and phase of plunge with respect to pitch) of the actual modes is denoted ○.

energy leading to flutter. This phenomenon is not obvious if the airspeed U is assumed constant while solving aeroelastic problems, as is usually done. With the assumption of constant velocity, it is implicit that whatever thrust required to maintain that velocity is provided by the engine. Thus, in a conventional aeroelastic analysis, the energy provided by the engine to the flow is missed.

Now, note that in actual flight the thrust may not be increased to maintain the flutter speed, but rather the thrust may be kept constant (as required for steady trimmed flight). If this is the case then, as the system starts to flutter, the drag on the aircraft will increase, and the aircraft will decelerate. It can be seen that the problem is now coupled between the airspeed and the oscillation amplitude. The airspeed determines the stability of the wing and, thus, the motion of the wing (whether it is damped or grows exponentially). On the other hand, the amplitude of oscillation determines the drag on the aircraft and, thus, the deceleration (or acceleration) of the aircraft. For further discussion on this topic, the reader is referred to Patil.[10]

Conclusions

The paper presents an energy-based paradigm that explains wing flutter and flapping flight in an integrated framework. Such a framework is helpful in recognizing the sources of energies involved in various aeroelastic phenomena and in understanding the corresponding energy transfer mechanisms. This integrated framework leads to a few conclusions.

1) There are three types of modes in an aeroelastic system, namely, 1) unstable mode with increase in drag (flutter mode), 2) stable mode with increase in drag, and 3) stable mode generating thrust (flapping flight mode).

2) The energy for flutter comes from the propulsive unit in the aircraft. It does not come from the flow, although the propulsive energy may be transferred to the structure via the flow.

3) Flutter always leads to drag, and thrust producing modes are always stable.

4) The flutter region (in the modal vector space) is separate from the thrust-producing region.

5) For a two-dimensional airfoil model, the flutter region and the thrust-producing region are tangential to one another, touching at a neutral point, where there is no energy transfer.

6) The flutter region decreases with reduced frequency, whereas the thrust-producing region increases.

7) The efficiency of flapping and the level of propulsive power generated depends on the mode shape.

8) The actual modes of an aeroelastic system move (with change in velocity) relative to the energy curves and may change from one type of mode (flutter with drag or stable with drag or stable with thrust) to another.

9) The drag induced due to oscillations at flutter will lead to complicated behavior in terms of the airspeed and the flutter oscillation amplitude.

Acknowledgments

This work was partially supported by the U.S. Air Force Office of Scientific Research (Grant number F49620-98-1-0032), Brian P. Sanders, Technical Monitor. The author would also like to acknowledge helpful discussions with Dewey Hodges from Georgia Institute of Technology.

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